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Embodied Transmission of Ideas: Collaborative Construction of Geometry Content and Mathematical Thinking

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Abstract: This case study looks at how students embody their ideas about geometry conjectures and how those ideas travel within and between student groups. In one classroom of a Title 1 high school, students participated in a three-part program in which they: (1) played *The Hidden Village*, a motion-capture video game where they assess the veracity of geometric conjectures (i.e., if it is always true or ever false) while their intuitions, insights, and rationales (including their gestures) are video recorded, (2) designed their own directed actions (i.e., a sequence of movements that represents a body-based interpretation of the structure and transformation of a spatial configuration), and (3) re-played the game with a mixture of previous conjectures combined with the conjectures designed by their peers. These two cases revealed ways that simulated enactment and collaborative construction can convey mathematical ideas.

Keywords: Embodiment, Geometry, Collaborative Construction, Transfer

Introduction

In a small pilot study with one classroom over three class periods, spread across three weeks, students played a motion-capture video game, *The Hidden Village,* and were provided opportunities to make new content for the game. In doing so, students were invited to think, act, and talk through the ways that their bodies could *represent* geometric objects in the conjectures--statements that are provable false or true--and how body movements could *enact* the conjectures' geometric transformations in the process of proof performance. We hypothesize that these embodied sequences, called *directed actions*, can foster mathematical insights crucial for students' understanding. This research evaluates how students, as designers of their own directed actions for geometric conjectures, express their conceptualizations using their bodies, not only to their fellow group members, but also how the consensus in their thinking as a group transmits when the conjecture they designed is played, and the directed actions performed, by other groups in the context of the game. Researchers analyzed students' subsequent production of simulated actions, as evidenced by the gestures they made in the process of exploring and explaining their thinking.

Theoretical Background

Studies have shown that mathematics can be learned through action-based interventions (Abrahamson & Sánchez-García, 2016; Smith, King & Hoyte, 2014). *The Hidden Village* (THV; Authors) is an educational video game. It draws on the theory of *Gesture as Simulated Action* (GSA; Hostetter & Alibali, 2019), which asserts that gestures activate perceptual-motor processes in the brain when co-articulated with speech or thought. These sensorimotor experiences can induce cognitive states through the process of *Action-Cognition Transduction* (ACT; Nathan, 2017). From this, Nathan and Walkington (2017) developed the *Grounded and Embodied Cognition* (GEC) framework, which proposed that directing players' bodily movements (via directed actions) will complement learners' verbal expressions of mathematical reasoning. THV provides a platform for learners to collaboratively engage with mathematical ideas through movement and speech, and video records their mathematical interactions.

An embodied theory of transfer (Alibali & Nathan, *in press*) posits that concepts are ultimately represented by the actions, gestures, and other body-based resources, embedded in various physical and social settings – like collaborative game play. This embodied transfer expands between individuals and across multiple modalities through the exchange of actual and simulated perception and action between and among individuals. We call this form of embodied transfer "travel." By prompting players to explain their answers, THV primes players' production of *dynamic depictive gestures* that mentally and physically simulate transformations of mathematical objects through multiple states (Garcia & Infante, 2012; Walkington et al., 2014), which fosters the generalizations needed for deductive proof and proof by contradiction. At the same time, the outward transmission of dynamic gestures are hypothesized to facilitate transfer to other players through travel.

This case study evaluates how other students interpreted and followed the directed actions created by their peers. It was used to investigate how these embodied mathematical ideas generated by one group "travelled" through player-generated content to other students through movements prompted by their subsequent game play. We explored two research questions: (RQ1) How does a student group designing new game content develop their mathematical ideas and create their own directed actions intended for others to play? (RQ2) How does the intention of the original group's mathematical ideas "travel" to other student groups through subsequent game play, and show up as the embodied transfer of those ideas in other groups' gesture and speech? Researchers investigate how student groups created directed actions for geometry conjectures and formed their ideas about geometric transformations.

Methods

Participants

In this study, 12 students in a Title I high school in the US participated in a three-day embodied mathematics curriculum focused on geometric thinking. Students were randomly assigned to groups of three or four. Although the groups were mostly consistent for the three days of the program, two students did not complete the entire curriculum, thus this paper focuses on the remaining student groups. Our analysis is anchored in Group 1, whose interest playing THV and constructive collaboration using THV conjecture editor produced a set of directed actions most depictive of the geometric transformation in their randomly assigned conjecture.

Procedure

Students took part in a three-day curriculum over three class periods over three successive weeks: (1) gameplay of a shortened version of the original THV (six conjectures from the original curriculum administered in previous studies (Authors, 2020), (2) the co-design activity of creating their own directed actions for a new conjecture, and (3) gameplay of a new THV curriculum (eight conjectures, three repeated from Day 1, three designed by all the student groups on Day 2, and two novel items). The in-situ curriculum was administered by the research team during normal class time, including worksheets for students to indicate their individual responses to the conjectures (i.e., their intuitions) along with video and audio recordings of students' group gameplay and co-design activities. These video recordings of the gameplay were then transcribed by researchers.

On Day 1, each student took a pretest surveying domain knowledge in geometry, spatial abilities, and demographics. Students were randomly assigned into groups of three or four and took turns playing individual conjectures within THV. For each turn, one student in a group performed the directed actions (i.e., the player), while the other members of the group watched (i.e., the observers). After performing directed actions, all members in a group were encouraged to engage in group discussion for the mathematical reasoning during gameplay.

On Day 2, the research team and teacher worked with student groups in the co-design activity. Researchers discussed the principles of embodied mathematics, and how directed actions in the game can connect body movements to the geometric transformations in the conjectures. Working in triads, students were given a new conjecture for which they created their own set of directed actions. Collaboratively, teams discussed and designed their own directed actions using the THV conjecture editor (see Figure 3). Students' group work is automatically uploaded to a shared database, accessible to other player groups.

On Day 3, student groups played a new module of THV that included conjectures previously played on Day 1, the new conjectures designed on Day 2, as well as and the directed actions created by themselves and by their peers with the conjectures. Similar to Day 1, students in a group played the game individually by taking turns and had a group discussion for the proofs.

Materials

The Hidden Village (THV) Game

THV is a 3D motion-capture collaborative video game that offers an immersive embodied geometry curriculum in which each player emulates in-game avatar's movements and then reasons about geometry conjectures to prove whether it is either ever false or always true. THV consists of 6 parts in which players (see Figure 1): (1) meet characters of the hidden village, (2) perform directed actions when mimicking an avatar's mathematically relevant movements, (3) voice their intuition as to the veracity of the conjecture's statement (i.e., false or always true), (4) explain their insights for their rationale through speech and gesture, (5) provide a multiple-choice that best explains the conjecture, and (6) receive a token and progress through the village towards the goal of returning home from the hidden village.

Figure 1. The overall structure of THV gameplay.

The Hidden Village (THV) Conjecture Editor

The THV Conjecture Editor enables students to create new movement-based game content for others. Students add new conjectures and then design mathematically relevant directed actions for players by manipulating the sequences of poses made by the avatar (Figure 2). Using the Pose Editor, triads collaboratively generate 2-3 poses (starting, intermediate, and target pose; middle panel of Figure 2) to create directed actions for each conjecture. Once poses have been designed, players can preview the movements as a GIF-like animation. The completed user-generated actions are stored in the online database of THV and accessible to any other users to access and play.

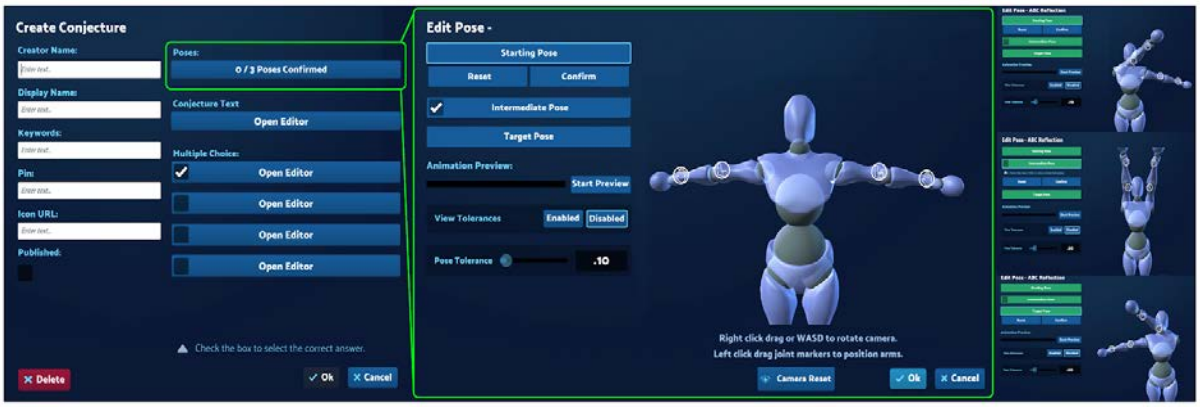

Figure 2. The THV Conjecture Editor and THV Pose Editor (for creating directed actions) with an example of a directed action sequence (far right).

Results

Within-Group Analysis

To understand learning processes in the co-design activity, researchers analyzed students' (Group 1) collaborative multimodal interactions during group discussions, including both gestural and verbal communication. Group 1 wPresented here is an example of students co-constructing directed actions for their chosen conjecture. Included is a photo transcript of Group 1's discussion and gestures while articulating their mathematical ideas (RQ1).

In the course of designing their directed actions, students used multiple dynamic depictive gestures (i.e., action-speech pairings, Authors, 2017) while deliberating which directed actions would best assist players to grasp the geometric relations relevant to proving their conjecture, the *ABC Reflection* (which is false)*: Given three points A, B, and C, and their reflected images about a line, A', B', and C', then ∠ABC and ∠A'B'C' are not equal.*

Transcript #1: (*N.B. S1 indicates Student #1; R indicates a researcher*; brackets [...] indicate gestures.)
[1] S1: Start? Ok. I have a question. We were talking about *ABC Reflection*. And how the two angles are
[2] congruent if it's flipped over the other axis. How will we be able to show that in this?
[3] R: What's a reflection?
[4] S1: [Flipped from their palm to the back of their hand; first over an imaginary y-axis, then over x-axis]

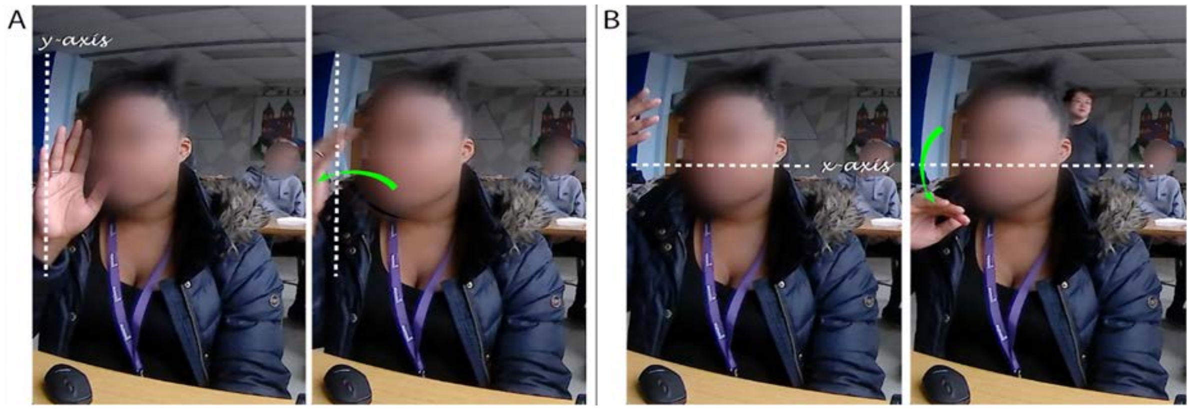

Figure 3. Student 1 in Group 1 shows the concept of reflection, flipping their hand over imaginary x and y axes.

In the beginning phase of the co-design activity, Student S1 asked a researcher how to show when an angle is "flipped over the other axis" (i.e., reflected) to represent that the two angles are congruent after the transformation. The researcher (R in Transcript #1) prompted S1 to consider the definition of a reflection as a scaffold for generating embodied mathematical ideas. In Line 4, S1 demonstrated the concept of reflection to the researcher in the form of a dynamic depictive gesture by flipping their hand over both an imaginary x and y-axis (see Figure 3).

Transcript #2: (*N.B. S2 indicates Student #2; R indicates a researcher*; brackets [...] indicate gestures.)
[1] S2: So like, the angle that we want to use is like this [forms ∠ABC using their arms]. And then if we flip it
[2] over, it'll be like this. [Rotates ∠ABC from the right side to the left side]
[3] R: I think that's a rotation if you move your arms like that. But what if I do like [flips hands over midline
[4] of the body]?
[5] All: Ohhhhh.
[6] R: Your body is the axis.

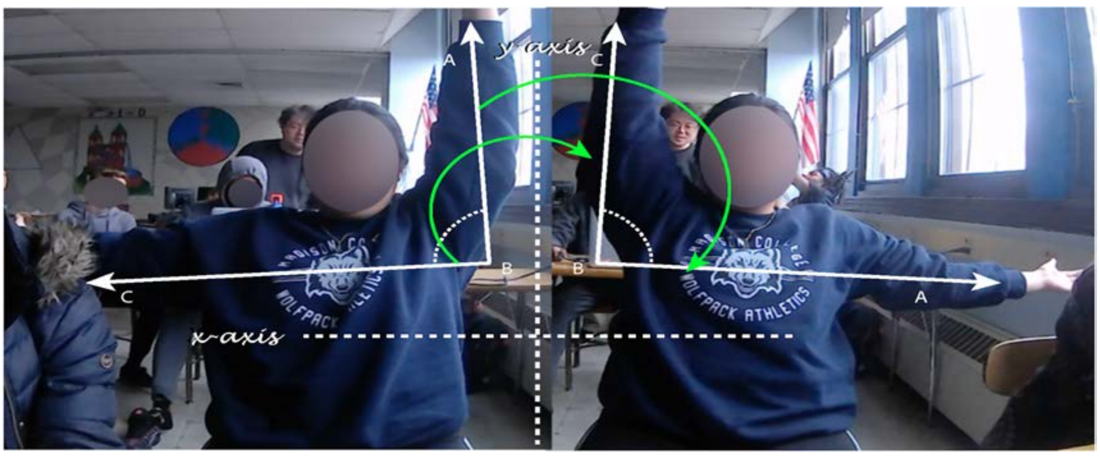

Figure 4. Student 2 in Group 1 suggests embodying the concept of an angle reflection that will "flip it over". S2's arm movements actually rotate the ∠ABC across an imaginary y-axis, transforming it to ∠CBA.

Student 2 (S2 in Transcript #2) suggested a directed action for the *ABC Reflection* conjecture, pointing their arms at an angle on the left side of their body, then rotating (not reflecting) them to the right side (see Figure 4). Noticing that the student has achieved the desired outcome of the conjecture, but through a logical but incorrect rotational transformation, the researcher scaffolded the student's insights by calling attention to the student's movements. The researcher suggested using the body as an axis for reflection. This helped students gain insight ("ohhh") and clarify their understandings of the geometric transformation, as Transcript #3 shows.

Transcript #3: (*N.B. S1 indicates Student #1;* brackets [...] indicate gestures.)
[1] S1: Oh, wait. This is not the starting pose. Is that the starting pose? [Uses arms to make ∠ABC on the left
[2] side of the body] We are going like, this is the angle [shifted arms directly to the right side of her
[3] body by performing a reflection across the body vertical axis]... Boom💥! That's the angle!

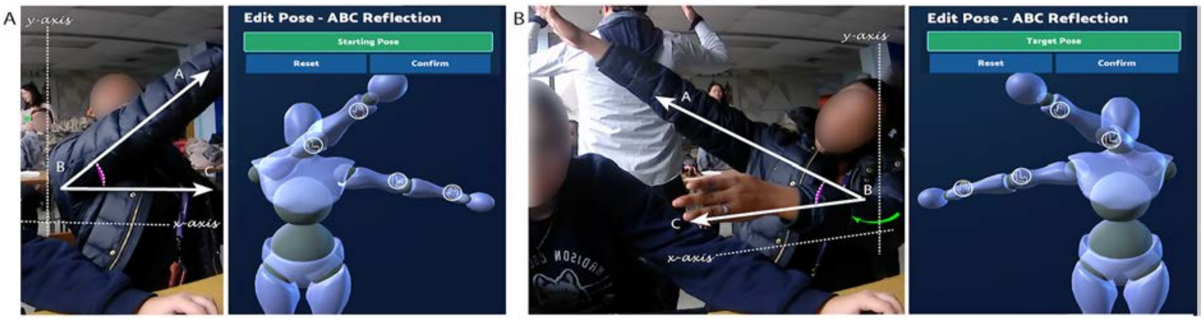

Figure 5. For *ABC Reflection* conjecture, Student 1 in Group 1 embodies the starting pose physically (also shown as designed in THV Pose Editor, panel A) and S1 performs the entire directed action, finishing on the target pose.

The third transcript indicates the starting and target poses (see far right panel of Figure 3) the student group used for the *ABC Reflection* conjecture. Narrating their actions as they shift the angle from the right side of the body to the left, S1 embodied the idea of "using your body as the midline" through this directed action. In finalizing these directed actions, the group members solidified their understanding of the conceptual difference between reflection and rotation in the process of designing their pose sequences in the THV pose editor (see Figure 5).

Between – group analysis

On Day 3, students played THV with a mixture of conjectures they had seen previously on Day 1, conjectures designed by their peers from Day 2, plus two previously unseen conjectures. Students in Groups 2 and 4 played the *ABC Reflection* conjecture as designed by Group 1. One player per group performed the directed actions prompted within the THV game, while the other group members observed. To track how Group 1's embodied mathematical ideas traveled to other groups (RQ2), researchers analyzed students' gestures and verbal discourse.

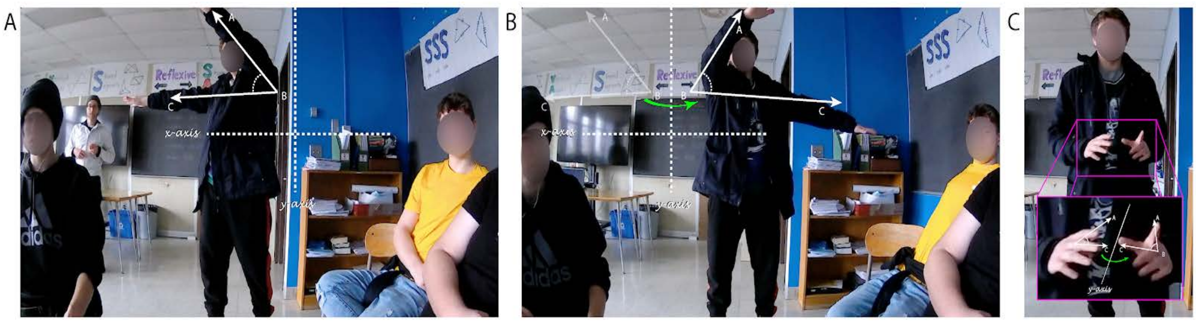

Figure 6. Student 3 in Group 4 performs the directed actions (Panels A & B) for *ABC Reflection*. In Panel C, S3 provides their intuition (TRUE or FALSE) and rationale, using their hands to represent the reflection of ∠ ABC

Transcript #4: (*N.B. S3 indicates Student #3;* brackets [...] indicate gestures.)
[1] S3: False. Because it can be proportionally the same, have the same angles [using hands to make an angle]
[2] while being in different locations. [S3 then, selects the correct answer from the multiple-choice options]

After performing the directed actions during game play (see Figure 6), Student 3 (Transcript #4) states their intuition ("False"). S3 provides their rationale (Lines [1-2]) with spontaneous gestures (see Figure 6, far right). In the process of proofing the conjecture, S3's spontaneous gesture demonstrates an embodied conceptualization of the ∠ ABC that results from the transformation. In effect, this truncated gesture complements S3's verbal rationale and extends Group 1's original idea for embodying the reflection of an angle over an axis.

Transcript #5: (*N.B. S4 and S5 indicates Students #4 and #5, respectively;* brackets [...] indicate gestures.)
[1] S4: [Re-reads the *ABC reflection* conjecture] What? They are not equal?
[2] S5: [Places hand on chin and leans forward, thinking about conjecture]
[3] S4: Do you know what to say about it?
[4] S5: [Shakes his head no]
[5] S4: I think it has something to do with reflection, but I don't know.
[6] S5: True.
[7] S4: Well. I don't know. It could be true, but it could also be not true.
…
[8] S4: I think it's reflected 'cause it said reflecting... I think it's reflected [flips the left hand over]. And,
[9] ummm, [reading the conjecture carefully] I don't know. Do C maybe. Oh wait no, do D. do D.
[10] S5: [Makes a pose to select the option C, then, changes the pose to select option D]

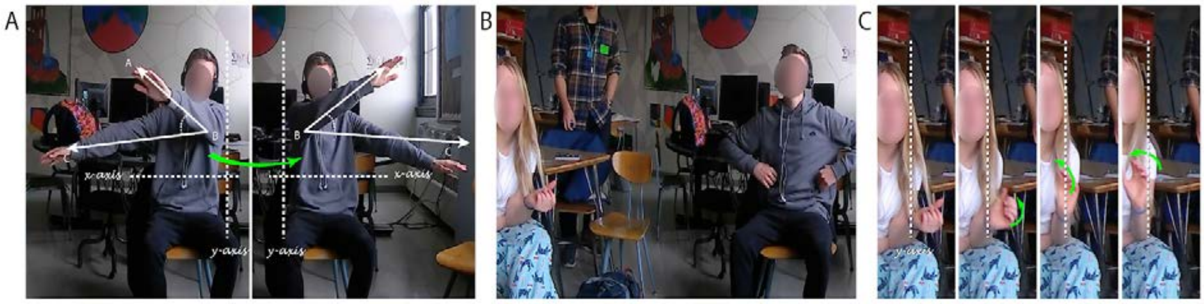

Figure 7. Student 5 (S5) in Group 2, performs the directed actions for the *ABC Reflection* conjecture created by Group 1 (Panel A). Student 4 (S4), an observer in Group 2, discusses the conjecture with S5 (Panel B), and in the process makes a spontaneous dynamic gesture, depicting the reflection over the y-axis.

In Transcript 5, Group 2 plays Group 1's *ABC Reflection* conjecture (see Figure 7, panel A & B). In this case, an observer (S4 in Transcript #5) actively reasons through the conjecture along with the player (S5). Student 4 inquires, "What? They are not equal?" (Line 1), consulting with Student 5, who does not provide any rationale. Student 4 focuses on the word "reflection", making a spontaneous gesture depicting the reflection of ∠ ABC. Recall that this was a key consideration for Group 1's design of the directed actions. By flipping over the hand to show the palm, Student 4 externalizes the concept of reflection and their understanding of the conjecture. With this insight, Student 4 initially instructs Student 5 to pick the correct answer, only interrupt that choice with a second-guess for Student 5 to pick a different, incorrect answer.

Discussion

This pilot study demonstrated how mathematical ideas "travel" through embodied actions. THV served as a vehicle to reify geometric relations as movements of an avatar. Students created content that coupled geometric conjectures with movements intended to help players to embody these mathematical ideas. We found that students used the posable avatar as a way to explore embodied ways of reasoning and then share those ideas through subsequent game play. Performing these directed actions helped new players build up their mathematical intuitions and articulate their multimodal justifications and transformational proofs. This work is limited to a single classroom. Still, it offers insights for how embodied interventions support travel of abstract concepts. We now revisit the research questions.

Research question 1 asked: How does a student group designing new game content develop their mathematical ideas and create their own directed actions intended for others to play? To address this, we analyzed ways students in Group 1 communicated their ideas to each other about the *ABC Reflection* conjecture and their discussion on the design of their directed actions. Collaborative discussion in Group 1 showed how students used embodiment to garner insights about angles and axes, and differentiate between rotation and reflection to understand the conjecture and work through multiple ideas for their directed actions. First, Student 1 ruminated over the meaning of reflection, articulating dynamic depictive gestures of reflections over the x- and y-axes. This prompted Student 2 to embody the ABC reflection, making a right angle with her body, that she subsequently rotated from right to left, to which the researcher highlighted the difference from the reflection in the conjecture. Moreover, Student 2's externalization of thought spurred Student 1 to reconsider the proposed directed action and propose a new movement that more accurately depicted a reflection of ∠ABC. This group's collaboration exemplified how embodying mathematical thinking traveled among members of a design team as they collaborated on activities for their classmates. The directed actions Group 1 designed were emblematic of the relevant geometric transformations.

Research question 2 asked: How does the intention of the original group's mathematical ideas "travel" to other student groups through subsequent game play, and show up as the embodied transfer of those ideas in other groups' gesture and speech? RQ2 led us to examine Group 4's and Group 2's performances on Group 1's *ABC Reflection* conjecture, and their interpretation of the geometric transformation depicted by the in-game directed actions. We found that Group 1's embodied mathematical ideas traveled successfully to the other groups whose explanations expounded upon the concept of reflecting an angle over an axis. In Group 4, Student 3's response was accompanied by a truncated spontaneous gesture, a type of marking (see Kirsh, 2010) to represent the outcome of the geometric transformation. Nathan et al.'s (2017) *Grounded Embodied Cognition* framework contends that the directed actions prime the sensorimotor stimulation that preceded Student 3's gestural reaffirmation that the reflected angle across the y-axis was indeed congruent. In Group 2, an observer (S4) shifted from a passive role to a more active role in the group's reasoning process (cf. Chi & Wylie's 2014 *ICAP* framework). In doing so, Student 4's contemplations and accompanying spontaneous dynamic gesture (Walkington et al., 2014) depicted the angle reflection over the y-axis. This contributed to Student 4's consultation with the group's current player (S5) in the discussion preceding their multiple-choice selection. Student 4's initial choice was correct and her decision to change her answer was most likely the result of over-thinking (Wilson & Schooler, 1991).

These case studies identify some of the promises of an embodied mathematics curriculum. Directed actions are a malleable factor that can scaffold cognition, and their historical traces, can give rise to successful performances of mathematical proofs, complete with spontaneous gestures coupled with task-relevant speech. This pilot study, which has since been expanded to a full intervention across all sections of geometry in a large Southern US public school. Here we can see ways these collaborative, embodied interactions offer a proof-of-concept of the feasibility of scaling THV to classroom use and as a resource for computer supported collaborative learning and transfer.

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